Exotic Localized Structures of the (2+1)-Dimensional Nizhnik-Novikov-Veselov System Obtained via the Extended Homogeneous Balance Method

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Z. Naturforsch. **61a,** 216 – 224 (2006); received March 27, 2006

In this paper, we successfully apply the extended homogeneous balance method (EHBM) to derive a new type of variable separation solutions for the (2+1)-dimensional Nizhnik-Novikov-Veselov system. Novel localized coherent structures about multi-valued functions, i.e., special dromion, special peakon and foldon, and the interactions among them are discussed. Moreover, the explicit phase shifts for all the local excitations offered by the quantity U are given and applied to novel interactions among special dromion, special peakon and foldon in detail. – PACS numbers: 05.45.Yv, 02.30.Jr, 02.03Ik

Key words: Exotic Localized Structures; (2+1)-Dimensional Nizhnik-Novikov-Veselov System; Extended Homogeneous Balance Method.

1. Introduction

(1+1)-Dimensional solitons and solitary wave solutions have been a topic of intense investigation in several branches of science such as condensed matter physics, fluid mechanics, plasma physics, optics, [1]. In (2+1) dimensions rich localized coherent structures, including dromion [2], peakon [3], compacton [4], foldon [5], ghoston [6], ring soliton [3], and the interaction between these solitons [3, 5, 7, 8] have been extensively discussed. Moreover, non-propagating solitons [9], chaotic and fractal patterns [3], periodic and quasi-periodic waves [10] have also been studied well. From the symmetry study of (2+1)-dimensional integrable models one knows that there exist much more abundant symmetry structures than in lower dimensions [11]. This fact suggests that the coherent structure of the (2+1)-dimensional integrable models may have quite rich structures that have not yet been revealed.

It is well-known that the interaction of solitons in (1+1)-dimensional nonlinear models is considered to be elastic. That means there is no exchange of any physical quantity like the energy and the momentum among interacting solitons. That means, except for the phase shifts, the shapes and velocities of solitons all remain unchanged. However, there exist more interactions among localized structures in (2+1)-dimensional nonlinear models, i.e., elastic and non-elastic interactions have both been investigated between various coherent localized structures (see [7] and references therein in detail). In [5], the single-valued structures, i. e., special dromion and peakon, have been constructed by selecting multi-valued functions. To our knowledge, the interactions between these single-valued structures constructed by multi-valued functions are still open questions. In order to discuss these exotic interactions more directly and visually, we take the (2+1)-dimensional nonlinear Nizhnik-Novikov-Veselov (NNV) equation as a concrete example. The system is of the form

$$u_t + au_{xxx} + bu_{yyy} - 3a(uv)_x - 3b(uw)_y = 0, \quad (1)$$

$$u_x = v_y, \quad u_y = w_x, \quad (2)$$

where a and b are arbitrary constants. This system is simply a known isotropic Lax extension of the well known (1+1)-dimensional Korteweg – de Vries (KdV) equation. Some types of the soliton solutions have been studied by many authors. For instance, Boiti, Leon, Manna and Pempinelli [12] solved the NNV equation via the inverse scattering transformation. Tagami [13] obtained the soliton-like solutions of the NNV equation by means of the Bäklund transformation. Hu and

Willox [14] also gave the nonlinear superposition formula of the NNV equation. Ohta [15] obtained the Pfaffian solutions for the NNV equation. Radha and Lakshmanan [16] constructed only its dromion solutions from its bilinear form after analyzing its integrability aspects. Lou [17] got some new special types of multisoliton solutions of the NNV equation by using the standard truncated Painlevé analysis.

The paper is arranged as follows. In Section 2, we obtain the variable separation solutions of the NNV system via the variable separation approach (VSA) based on the extended homogeneous balance method (EHBM) [7, 18]. Three kinds of coherent localized structures and interactions among them are discussed analytically and graphically in Section 3. Finally, a short summary is presented.

2. Variable Separation Solutions for the (2+1)-Dimensional NNV System

According to the idea of EHBM, considering the balance in part between the nonlinear terms and the higher order partial derivative terms in the system (1) and (2), we suppose that its solution is of the form

$$u = f(\varphi)_{xy} + u_0, \tag{3}$$

$$v = g(\varphi)_{xx} + v_0, \tag{4}$$

$$w = f(\varphi)_{yy} + w_0, \tag{5}$$

where $f(\varphi), g(\varphi), w(\varphi)$ and $\varphi(x,y,t)$ are to be determined later, and u_0, v_0, w_0 are arbitrary known seed solutions. It is evident that (1) and (2) possess trivial seed solutions $u_0 = 0, v_0 = v_0(x,t), w_0 = w_0(y,t)$. Now substituting (3)–(5) together with the seed solutions into (1) and (2) yields

$$\begin{aligned} &u_t + au_{xxx} + bu_{yyy} - 3a(uv)_x - 3b(uw)_y = \\ &(f^{(5)} - 3f''g^{(3)} - 3f^{(3)}g'')a\varphi_x^4\varphi_y \\ &+(f^{(5)} - 3f''h^{(3)} - 3f^{(3)}h'')b\varphi_x\varphi_y^4 \\ &+ \text{lower power terms of the derivatives of} \\ &\quad \varphi(x,y,t) \text{ with respect to } \{x,y,t\} = 0, \end{aligned} \tag{6}$$

$$\begin{aligned} &(f^{(3)} - g^{(3)})\varphi_x^2\varphi_y + (f'' - g'')\varphi_{xx}\varphi_y \\ &+2(f'' - g'')\varphi_x\varphi_{xy} + (f' - g')\varphi_{xxy} = 0, \end{aligned} \tag{7}$$

$$\begin{aligned} &(f^{(3)} - h^{(3)})\varphi_x\varphi_y^2 + (f'' - h'')\varphi_x\varphi_{yy} \\ &+2(f'' - g'')\varphi_y\varphi_{xy} + (f' - g')\varphi_{xyy} = 0. \end{aligned} \tag{8}$$

Setting the coefficients of the terms with $a\varphi_x^4\varphi_y$ and $b\varphi_x\varphi_y^4$ in (6), $\varphi_x^2\varphi_y$ in (7) and $\varphi_x\varphi_y^2$ in (8) to zero, we obtain the following ordinary differential equations for the functions $f(\varphi)$, $g(\varphi)$ and $h(\varphi)$:

$$f^{(5)} - 3f''g^{(3)} - 3f^{(3)}g'' = 0, \tag{9}$$

$$f^{(5)} - 3f''h^{(3)} - 3f^{(3)}h'' = 0, \tag{10}$$

$$f^{(3)} - g^{(3)} = 0, \tag{11}$$

$$f^{(3)} - h^{(3)} = 0. \tag{12}$$

The following special solutions exist for (9)–(12):

$$f(\varphi) = g(\varphi) = h(\varphi) = -2\ln(\varphi). \tag{13}$$

Therefore,

$$\begin{aligned} &f'f'' = f^{(3)}, \quad f'^2 = 2f'', \quad f''^2 = \frac{1}{3}f^{(4)}, \\ &f'f^{(3)} = \frac{2}{3}f^{(4)}, \quad f''f^{(3)} = \frac{1}{6}f^{(5)}. \end{aligned} \tag{14}$$

Using (13) and (14), (10) and (11) are satisfied automatically, and (9) can be simplified as

$$\begin{aligned} &[(\varphi_t + a\varphi_{xxx} + b\varphi_{yyy} - 3av_0\varphi_x - 3bw_0\varphi_y)\varphi_x\varphi_y \\ &+3(a\varphi_x\varphi_{xx}\varphi_{xy} + b\varphi_y\varphi_{yy}\varphi_{xy} - a\varphi_x^2\varphi_{xxy} - b\varphi_y^2\varphi_{xyy})]f^{(3)} \\ &+[(\varphi_t + a\varphi_{xxx} + b\varphi_{yyy} - 3av_0\varphi_x - 3bw_0\varphi_y)_x\varphi_y \\ &+(\varphi_t + a\varphi_{xxx} + b\varphi_{yyy} - 3av_0\varphi_x - 3bw_0\varphi_y)_y\varphi_x \\ &+(\varphi_t + a\varphi_{xxx} + b\varphi_{yyy} - 3av_0\varphi_x - 3bw_0\varphi_y)\varphi_{xy} \\ &+3(-a\varphi_{xxx}\varphi_{xy} + a\varphi_x\varphi_{xxxy} - b\varphi_{xy}\varphi_{yyy} + b\varphi_y\varphi_{xyyy})]f'' \\ &+(\varphi_t + a\varphi_{xxx} + b\varphi_{yyy} - 3av_0\varphi_x - 3bw_0\varphi_y)_{xy}f' = 0. \end{aligned} \tag{15}$$

Setting the coefficients of $f^{(3)}, f''$ and f' in (15) to zero yields a set of partial differential equations for $\varphi(x,y,t)$:

$$\varphi_t + a\varphi_{xxx} + b\varphi_{yyy} - 3av_0\varphi_x - 3bw_0\varphi_y = 0, \tag{16}$$

$$a\varphi_x\varphi_{xx}\varphi_{xy} + b\varphi_y\varphi_{yy}\varphi_{xy} - a\varphi_x^2\varphi_{xxy} - b\varphi_y^2\varphi_{xyy} = 0, \tag{17}$$

$$-a\varphi_{xxx}\varphi_{xy} + a\varphi_x\varphi_{xxxy} - b\varphi_{xy}\varphi_{yyy} + b\varphi_y\varphi_{xyyy} = 0. \tag{18}$$

Because $v_0(x,t)$ and $w_0(y,t)$ are arbitrary functions of the variables $\{x,t\}$ and $\{y,t\}$, respectively, in (16)–(18) we can select an appropriate variable-separated hypothesis for the function $\varphi(x,y,t)$ as follows:

$$\varphi(x,y,t) = p(x,t) + q(y,t), \tag{19}$$

where $p(x,t)$ is an arbitrary function of variables $\{x,t\}$, $q(y,t)$ is an arbitrary function of variables $\{y,t\}$. Inserting (13) and (19) into (3)–(5), along with (16)–(18), and carrying out some careful and tedious calculations, we deduce a rather general exact solitary wave solution of the (2+1)-dimensional NNV equation in the form

$$u(x,y,t) = \frac{2p_x q_y}{(p+q)^2}, \tag{20}$$

$$v(x,y,t) = \frac{2p_x^2}{(p+q)^2} - \frac{2p_{xx}}{p+q} + \frac{p_t + a p_{xxx}}{3a p_x}, \tag{21}$$

$$w(x,y,t) = \frac{2q_y^2}{(p+q)^2} - \frac{2q_{yy}}{p+q} + \frac{q_t + b q_{yyy}}{3b q_y}, \tag{22}$$

with two arbitrary functions $p(x,t)$ and $q(y,t)$.

Remark 1: If setting $p(x,t) \equiv \frac{1+a_1 P(x,t)}{a_2+AP(x,t)}$, $q(y,t) \equiv Q(y,t)$ in the solution (20), the solution (12) in [17] can be recovered. This fact implies that we can substitute the complicated variable separation forms $1+a_1P+a_2Q+APQ$ in [17] as the simple and direct one $p+q$, which will greatly simplify the operation.

Remark 2: If taking $p(x,t) \equiv -\frac{1}{P(x,t)}$, $q(y,t) \equiv Q(y,t)$, the solution (20) changes into $u(x,y,t) = \frac{2P_xQ_y}{(1-PQ)^2}$. This fact suggests that the variable-separated ansatz $1-PQ$ is identical to the one $p+q$.

3. Exotic Localized Structures

In this section, we will discuss some special types of interesting localized structures for the quantity

$$U \equiv \frac{u}{2} \equiv \frac{p_x q_y}{(p+q)^2}, \tag{23}$$

from which we know that for general choices of p and q there may be some singularities for the quantity U. However, when the arbitrary functions p and q are chosen appropriately to avoid the singularities, there may exist abundant excitations for U. All rich localized coherent structures, such as non-propagating solitons, dromion, peakon, compacton, foldon, instanton, ghoston, ring soliton, and the interaction between these solitons, can be re-derived by using the quantities U expressed by (23). Moreover, if p or q is considered to be a periodic function or a solution of a chaos system like the Lorenz chaos system, then solitons possess periodic or chaotic behavior. It is well-known that there are some lower-dimensional stochastic fractal functions, which may be used to construct higher-dimensional stochastic fractal dromion and lump excitations by using the quantity U expressed by (23). Since these similar situations have been widely discussed [2–10, 17], the related plots are omitted in our present paper. Here we focus on some novel localized coherent structures of multi-valued functions and their interactions.

3.1. Special Dromion, Special Peakon and Foldon

From the presentation (23), dromions, peakons and foldons can be obtained by choosing p or q as a single-valued function, piecewise defined function, and a multi-valued function, respectively. However, in fact, these single-valued structures, i. e., dromions and peakons, can also be derived by selecting p or q as multi-valued function [5]. Based on the quantity (23), special dromions, special peakons and foldons can be constructed if we choose both p and q as the following series:

$$p_x = \sum_{i=1}^{N} \kappa_i(\zeta - c_i t), \quad x = \zeta + \sum_{i=1}^{N} \chi_i(\zeta - c_i t), \tag{24}$$

$$q_y = \sum_{j=1}^{M} \vartheta_j(\eta - d_j t), \quad y = \eta + \sum_{j=1}^{M} \lambda_j(\eta - d_j t), \tag{25}$$

where c_i $(i = 1,2,\ldots,N)$ and d_j $(j = 1,2,\ldots,M)$ are arbitrary integers, κ_i and χ_i, ϑ_j and λ_j are localized excitations with the properties $\kappa_i(\pm\infty) = 0$, $\chi_i(\pm\infty) =$ consts, $\vartheta_j(\pm\infty) = 0$, $\lambda_j(\pm\infty) =$ consts. From (24) and (25), one can show that ζ (or η) may be a multi-valued function in some suitable regions of x (or y) by choosing the functions χ_i (or λ_j) appropriately. Therefore, the function p_x (or q_y), which is obviously an interaction solution of N(or M) localized excitations due to the property $\zeta|_{x\to\infty} \to \infty$ (or $\eta|_{y\to\infty} \to \infty$), may be a multi-valued function of x (or y) in these areas, though it is a single-valued function of ζ (or η).

Specifically, p and q are chosen as

$$p_x = \operatorname{sech}^2(\zeta - t), \quad x = \zeta - A\tanh(\zeta - t), \tag{26}$$

$$q_y = \operatorname{sech}^2(\eta - t), \quad y = \eta - C\tanh(\eta - t), \tag{27}$$

where A and C are characteristic parameters, whose differences imply the different localized structures. When their values are chosen between 0 and 0.9, 0.9 and 1, and bigger than 1, special dromion, special peakon and foldon can be derived. Figure 1 describes three localized structures (special dromion, special peakon and foldon) with $A = C = 0.05, 0.95, 1.5$, respectively.

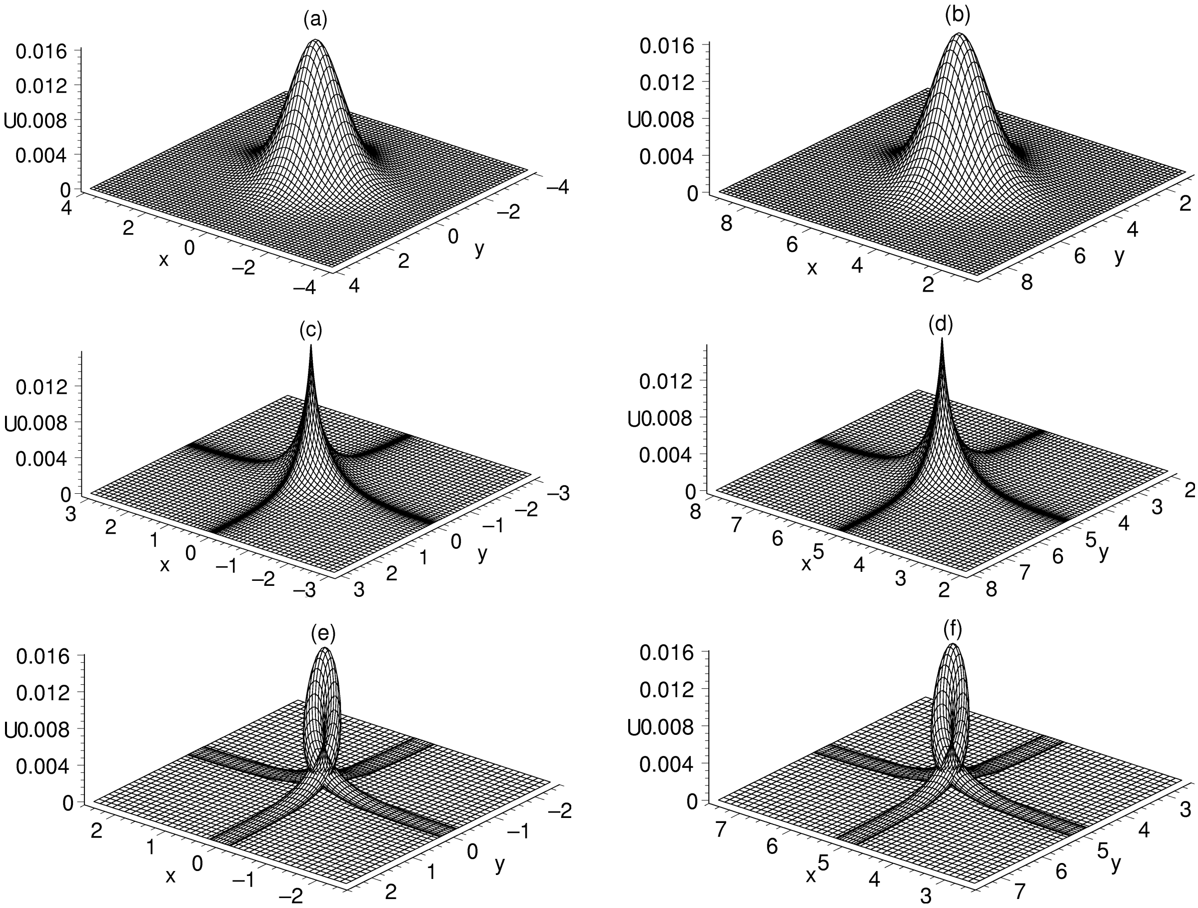


Fig. 1. (a), (b) Special dromion, (c), (d) special peakon, and (e), (f) foldon structures for U with conditions (26) and (27); (a), (b) $A = C = 0.05$; (c), (d) $A = C = 0.95$; (e), (f) $A = C = 1.5$ at $t = 0$ and $t = 5$, respectively.

3.2. Asymptotic Behaviors of the Localized Excitations Produced from (23)

The interaction can be elastic or inelastic. It is called elastic, if the amplitude, velocity and wave shape of solitons do not change after their interaction. Otherwise, the interaction between solitons is inelastic (incomplete elastic and completely inelastic). Like the collisions between two classical particles, a collision in which the solitons stick together is sometimes called completely inelastic, which is discussed in [8]. In order to discuss the interaction property of these localized excitations related to the expression (23), we first study the asymptotic behaviors of the localized excitations produced from the quantity (23) when $t \to \infty$.

In general, if the functions p and q [considering (24) and (25)] are chosen as multi-localized solitonic excitations with ($z_i \equiv \zeta - c_i t$, $Z_j \equiv \eta - d_j t$)

$$p|_{t\to\mp\infty} = \sum_{i=1}^{N} p_i^{\mp}, p_i^{\mp}(z_i) \equiv p_i(\zeta - c_i t) \equiv \int \kappa_i \mathrm{d}x|_{z_i\to\mp\infty}, \tag{28}$$

$$q|_{t\to\mp\infty} = \sum_{j=1}^{M} q_j^{\mp}, q_j^{\mp}(Z_j) \equiv q_j(\eta - d_j t) \equiv \int \vartheta_j \mathrm{d}y|_{Z_j\to\mp\infty}, \tag{29}$$

where $\{p_i, q_j\}\forall i$ and j are localized functions, then the physical quantity expressed by (23) delivers $M \times N$

(2+1)-dimensional localized excitations with the asymptotic behavior

$$U|_{t\to\mp\infty} \to \sum_{i=1}^{N}\sum_{j=1}^{M} \frac{p^{\mp}_{iz_i} q^{\mp}_{jZ_j}}{(1+\chi^{\mp}_{iz_i})(1+\lambda^{\mp}_{jZ_j})[(p^{\mp}_i(z_i)+\tilde{p}^{\mp}_i)+(q^{\mp}_j(Z_j)+\tilde{q}^{\mp}_j)]^2} \equiv \sum_{i=1}^{N}\sum_{j=1}^{M} U^{\mp}_{ij}, \tag{30}$$

$$x|_{t\to\mp\infty} \to \zeta + \delta^{\mp}_i + \chi^{\mp}_i(z_i), \tag{31}$$

$$y|_{t\to\mp\infty} \to \eta + \Delta^{\mp}_j + \lambda^{\mp}_j(Z_j), \tag{32}$$

with

$$\tilde{p}^{\mp}_i = \sum_{j<i} p_j(\mp\infty) + \sum_{j>i} p_j(\pm\infty), \tag{33}$$

$$\tilde{q}^{\mp}_i = \sum_{j<i} q_j(\mp\infty) + \sum_{j>i} q_j(\pm\infty), \tag{34}$$

$$\delta^{\mp}_i = \sum_{j<i} \chi_j(\mp\infty) + \sum_{j>i} \chi_j(\pm\infty), \tag{35}$$

$$\Delta^{\mp}_j = \sum_{j<i} \lambda_j(\mp\infty) + \sum_{j>i} \lambda_j(\pm\infty). \tag{36}$$

In the above discussion, it has been assumed, without loss of generality, that $c_i > c_j, d_i > d_j$ if $i > j$. From the asymptotic result (30), we discover some important and interesting facts.

(i) The ij-th localized excitation U_{ij} is a travelling wave moving with the velocity c_i along the positive $(c_i > 0)$ or negative $(c_i < 0)$ x direction, and d_j along the positive $(d_j > 0)$ or negative $(d_j < 0)$ y direction.

(ii) The properties of the ij-th localized excitation U_{ij} is only determined by p_i of (28) and q_j of (29).

(iii) The shape of the ij-th localized excitation U_{ij} will be changed (incomplete elastic or completely inelastic interaction) if

$$\tilde{p}^{+}_i \neq \tilde{p}^{-}_i, \tag{37}$$

and (or)

$$\tilde{q}^{+}_j \neq \tilde{q}^{-}_j, \tag{38}$$

following the interaction. On the contrary, it will preserve its shape (completely elastic interaction) during the interaction if

$$\tilde{p}^{+}_i = \tilde{p}^{-}_i, \tag{39}$$

$$\tilde{q}^{+}_j = \tilde{q}^{-}_j. \tag{40}$$

(iv) The phase shift of the ij-th localized excitation U_{ij} reads $\delta^{+}_i - \delta^{-}_i$ in the x direction and $\Delta^{+}_j - \Delta^{-}_j$ in the y direction.

3.3. Interactions among the Localized Coherent Excitations Produced by Multi-Valued Functions

Now we discuss some novel coherent structures for the expression U, and focus our attention on interactions among the localized coherent excitations produced by multi-valued functions. In Section 3.1, we present three interesting coherent excitations, i.e., special dromion, special peakon and foldon. Here we discuss some novel interactions among them. If we take the specific choice $N = 2$, $M = 2$, $c_1 = 0.5$, $c_2 = 0$, $d_1 = 0.5$, $d_2 = 0$ in (28) and (29), one has

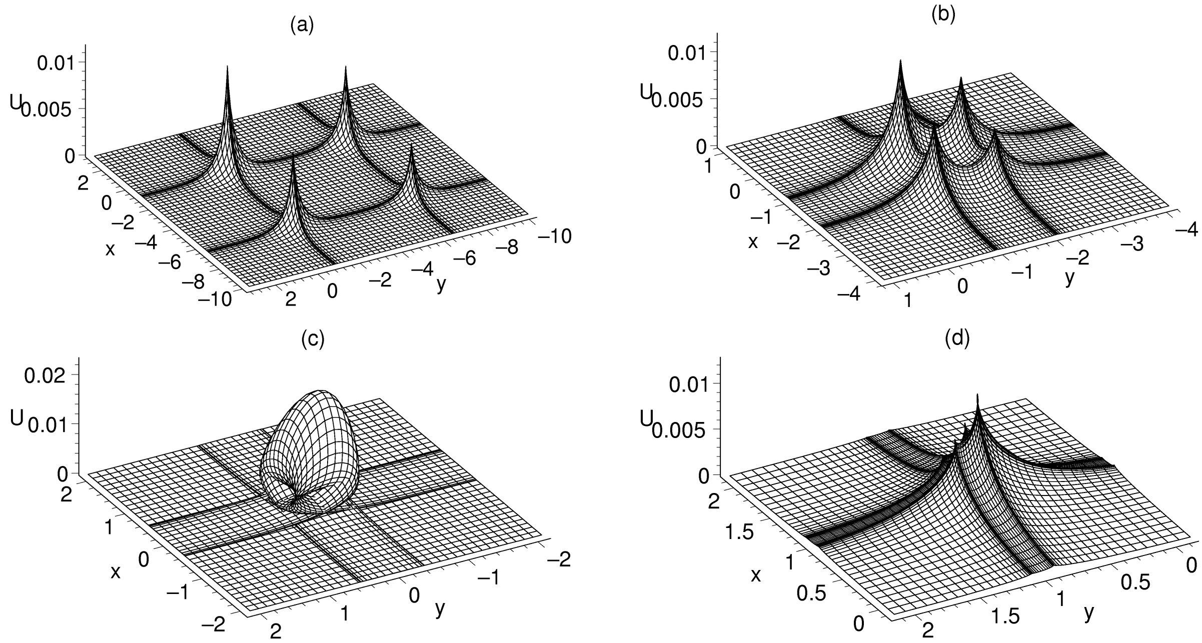
$$\begin{aligned} p_x &= 0.5\mathrm{sech}^2(\zeta - 0.5t) + 0.8\mathrm{sech}^2(\zeta), \\ x &= \zeta - A\tanh(\zeta - 0.5t) - B\tanh(\zeta), \end{aligned} \tag{41}$$

$$\begin{aligned} q_y &= 0.5\mathrm{sech}^2(\eta - 0.5t) + 0.8\mathrm{sech}^2(\eta), \\ y &= \eta - C\tanh(\eta - 0.5t) - D\tanh(\eta), \end{aligned} \tag{42}$$

where A, B, C and D are characteristic parameters, which determine the types of interaction and the phase shift of solitons. From the expression U with (41) and (42), one can obtain four solitons, one of which is static, and the others are moving. Generally, the phase shift of the static soliton is $\delta^{+}_2 - \delta^{-}_2 = \chi_1(-\infty) - \chi_1(+\infty) = 2A$ in x direction and $\Delta^{+}_2 - \Delta^{-}_2 = \lambda_1(-\infty) - \lambda_1(+\infty) = 2C$ in y direction, the phase shift of the moving smallest soliton is $\delta^{+}_1 - \delta^{-}_1 = \chi_2(-\infty) - \chi_2(+\infty) = -2B$ in x direction and $\Delta^{+}_1 - \Delta^{-}_1 = \lambda_2(-\infty) - \lambda_2(+\infty) = -2D$ in y direction.

Incomplete Elastic Interaction among Special Peakons

When we fix the values $A = B = C = D = 0.95$ in (41) and (42), we can successfully construct interactions among special peakons that possess phase shifts for the quantity U as depicted in Figure 2. From Fig. 2, we can see that the four special peakon localized excitations possess novel properties, that is, it is incomplete elastic since their shapes are not completely preserved after interaction and there also exists a multi-valued foldon in the process of their collision. Actually, the completely elastic interaction conditions (39) and (40) are not satisfied for the physical quantity (23) with (41)



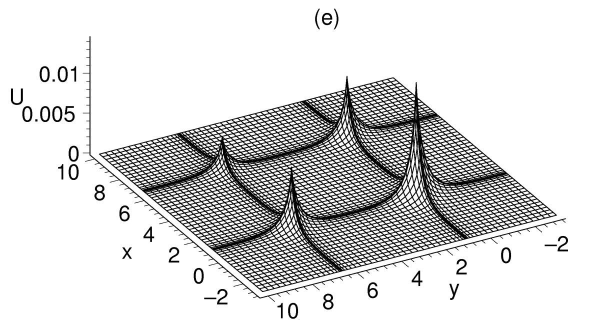


Fig. 2. Incomplete elastic interaction among special peakons for U with conditions (41) and (42) and $A = B = C = D = 0.95$ at time (a) $t = -15$; (b) $t = -6$; (c) $t = 0.2$; (d) $t = 4$; (e) $t = 15$.

and (42), i. e.,

$$\tilde{p}_2^+ - \tilde{p}_2^- = p_1(+\infty) - p_1(-\infty) = \frac{11}{30} \neq 0, \quad (43)$$

$$\tilde{p}_1^+ - \tilde{p}_1^- = p_2(-\infty) - p_2(+\infty) = -\frac{44}{75} \neq 0, \quad (44)$$

$$\tilde{q}_2^+ - \tilde{q}_2^- = q_1(+\infty) - q_1(-\infty) = \frac{11}{30} \neq 0, \quad (45)$$

$$\tilde{q}_1^+ - \tilde{q}_1^- = q_2(-\infty) - q_2(+\infty) = -\frac{44}{75} \neq 0. \quad (46)$$

In order to reveal the phase shift more clearly and visually, it has proved to be convenient and sufficient to fix one peakon possessing zero velocity. The phase shift can also be observed. As can be seen from Figs. 2a to e, before the interaction the static largest peakon is located at $\{x = -0.95, y = -0.95\}$, the moving smallest peakon is situated at $\{x = t + 0.95, y = t + 0.95\}$, while for the other two peakons, they are static in one direction and moving in the other direction and their centers are located at $\{x = -0.95, y = t + 0.95\}$ and $\{x = t + 0.95, y = -0.95\}$, respectively. After the interaction, the static peakon remains static and its center shifts to $\{x = 0.95, y = 0.95\}$, the smallest peakon shifts to $\{x = t - 0.95, y = t - 0.95\}$, the others have their centers shifted to $\{x = 0.95, y = t - 0.95\}$ and $\{x = t - 0.95, y = 0.95\}$, respectively. Therefore the phase shift of the static largest peakon is $\delta_2^+ - \delta_2^- = \chi_1(-\infty) - \chi_1(+\infty) = 1.9$ in x direction and $\Delta_2^+ - \Delta_2^- = \lambda_1(-\infty) - \lambda_1(+\infty) = 1.9$ in y direction, the phase shift of the moving smallest peakon is $\delta_1^+ - \delta_1^- = \chi_2(-\infty) - \chi_2(+\infty) = -1.9$ in x direction and $\Delta_1^+ - \Delta_1^- = \lambda_2(-\infty) - \lambda_2(+\infty) = -1.9$ in y direction.

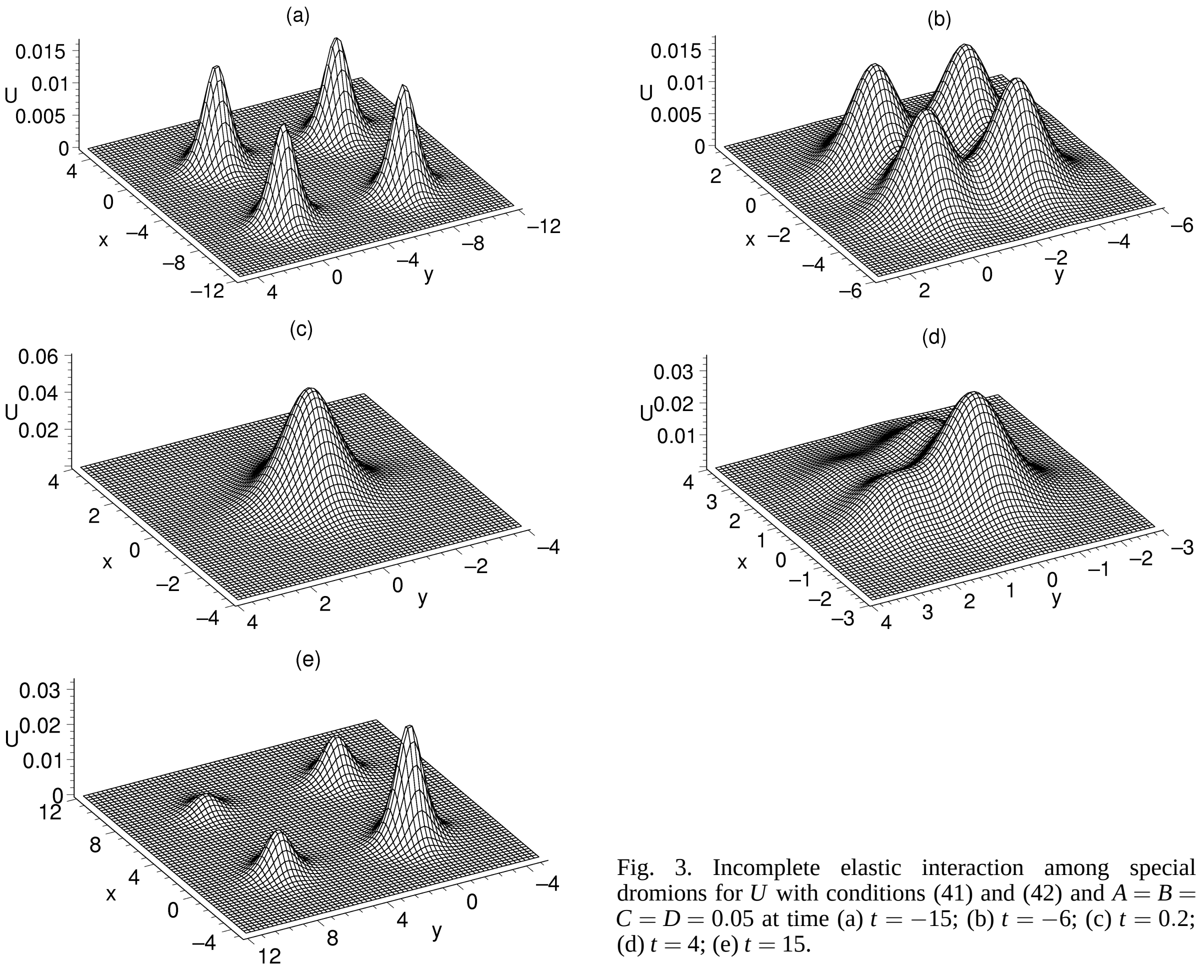


Fig. 3. Incomplete elastic interaction among special dromions for U with conditions (41) and (42) and $A = B = C = D = 0.05$ at time (a) $t = -15$; (b) $t = -6$; (c) $t = 0.2$; (d) $t = 4$; (e) $t = 15$.

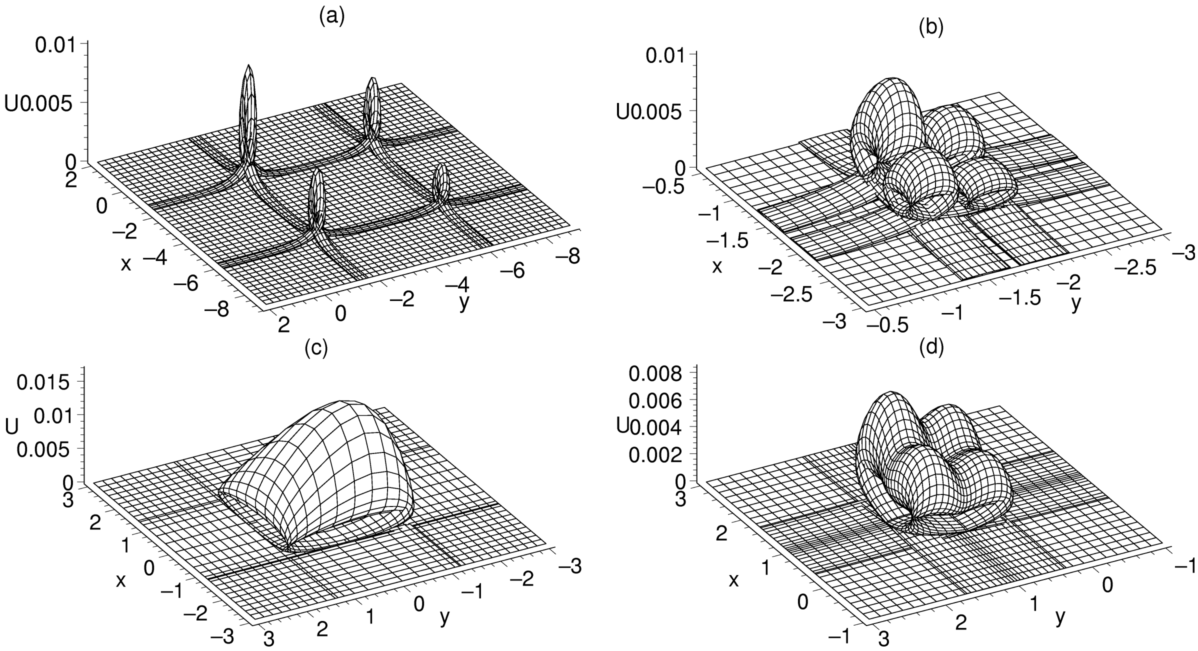
Incomplete Elastic Interaction among Special Dromions

If we take the specific values $A = B = C = D = 0.05$ in (41) and (42), then we successfully construct interaction among special dromions that possess phase shifts for the quantity U depicted in Figure 3. From Fig. 3, we can see that the interaction among the four special dromion localized excitations may exhibit a novel property, which is incomplete elastic since their shapes are not completely preserved after interaction. The analytical analysis of the incomplete interaction is similar to the previous case, that is, $\tilde{p}_2^+ - \tilde{p}_2^- = \frac{11}{30} \neq 0,\ \tilde{p}_1^+ - \tilde{p}_1^- = -\frac{44}{75} \neq 0,\ \tilde{q}_2^+ - \tilde{q}_2^- = \frac{11}{30} \neq 0,\ \tilde{q}_1^+ - \tilde{q}_1^- = -\frac{44}{75} \neq 0$. They do not satisfy the completely elastic interaction condition (39) and (40). A phase shift can also be observed. Similar to the analysis in the previous case, we can obtain the phase shift of these dromions, here we omit it for the limit of length.

Completely Elastic Interactions among Foldons

It is interesting to note that although the above choices result in incomplete elastic interaction behaviors for the (2+1)-dimensional solutions. We can also construct localized coherent structures with completely elastic interaction behaviors by selecting the values of A, B, C and D suitably in (41) and (42).

Along the above ideas and performing a similar analysis, if $A = B = C = D = 1.5$ in (41) and (42), interactions among foldons can be constructed for the physical quantity U depicted in Figure 4. The phase shift can also be observed, which is similar to the analysis before. Prior to interaction, the largest foldon



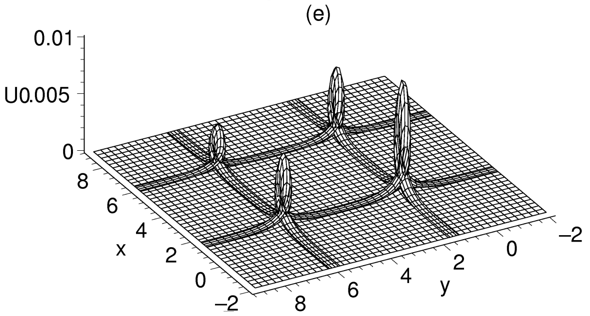


Fig. 4. Completely elastic interaction among foldons for U with conditions (41) and (42) and $A=B=C=D=1.5$ at time (a) $t=-15$; (b) $t=-7$; (c) $t=0.2$; (d) $t=4$; (e) $t=15$.

has set to be $\{v_{0x}=c_2=0, v_{0y}=d_2=0\}$, however, the position of the foldon is still changed from $\{x=-1.5, y=-1.5\}$ to $\{x=1.5, y=1.5\}$, then stops at $\{x=1.5, y=1.5\}$ and preserves its shape and initial velocities $\{v_x=v_{0x}, v_y=v_{0y}\}$ after interaction. Therefore the phase shift of the static largest foldon is $\delta_2^+-\delta_2^-=\chi_1(-\infty)-\chi_1(+\infty)=3$ in x direction and $\Delta_2^+-\Delta_2^-=\lambda_1(-\infty)-\lambda_1(+\infty)=3$ in y direction. The final velocities V_x and V_y of the moving smallest foldon are also the same as the initial velocities $\{V_x=V_{0x}=c_1=0.5, V_y=V_{0y}=d_1=0.5\}$. The phase shift of the moving smallest foldon is $\delta_1^+-\delta_1^-=\chi_2(+\infty)-\chi_2(-\infty)=-3$ in x direction and $\Delta_1^+-\Delta_1^-=\lambda_2(-\infty)-\lambda_2(+\infty)=-3$ in y direction. Moreover, from these evolution profiles and through detailed analysis, one can observe that they are completely elastic, which is very similar to the completely elastic collisions between two classical particles, since their shapes, amplitudes and velocities are completely preserved after interaction. Analytically, from (33), (34), (41) and (42), we have $\tilde{p}_2^+-\tilde{p}_2^-=0$, $\tilde{p}_1^+-\tilde{p}_1^-=0$, $\tilde{q}_2^+-\tilde{q}_2^-=0$, $\tilde{q}_1^+-\tilde{q}_1^-=0$. That is to say, the completely elastic interaction conditions (39) and (40) are really satisfied.

4. Summary and Discussion

In summary, the EHBM is applied to obtain variable separated solutions of the (2+1)-dimensional NNV system. Some lower dimensional arbitrary functions can be included in the exact solutions. Based on the quantity (23), three kinds of exotic coherent localized structures, that is, special dromion, special peakon and foldon, are discussed, and the interactions among them are investigated both analytically and graphically. Some novel properties and interesting behaviours are shown: the interactions among four foldons are completely elastic and possess phase shifts, and the interactions among four special dromions or four special peakons are incompletely elastic depending on the specific details of the solutions. Moreover, there exists a multi-valued foldon in the process of interactions among the four special peakons, which is reported here for the first time, to the best of our knowledge. Furthermore, the explicit phase shifts for all the local excitations offered by expression (23) given and applied to these novel interactions in detail. Of course, there are some pending issues to be further studied: How to quantify the notion of complete or incomplete elasticity more suitably besides analysis of asymptotic behaviors (37) – (40)? What is the measure for the deviation of a solution from elasticity? What is the general equation for the distribution of the energy and momentum for these exotic interactions?

In our future work, on the one hand, we will study how to generalize this method to other (2+1)-dimensional and (3+1)-dimensional nonlinear systems, and to differential-difference equations. We will also look for more interesting localized excitations.

Acknowledgement

The authors express their sincere thanks to the editors and the anonymous referees for their constructive suggestions and kind help.

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